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Simple Pendulum Experiment: Angular Approximation Revisited

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ABSTRACT

The acceleration of free fall should be realised by students as dependent on one's location rather than as a constant in all locations. Thus, there is no one way of determining g in the laboratory. For ease of science teaching it is important to let students know that one may not need to use small angular displacement of the pendulum bob in order to obtain g to an accuracy of three significant figures, comparable to, for example, the universally acceptable value of g = 9.81 ms^{-2} in London. A simple mathematical derivation enabled the determination of g = 9.93 ms^{-2}, which was 1.2 % higher than the universally accepted value.

Keywords: Angular Displacement, Free Fall, Gravity, Pendulum, Period

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I. INTRODUCTION

Traditionally, laboratory determination of acceleration of free fall (g) has used a number of methods that assume that angular displacements should be small. The literature shows that acceleration of free fall of a body depends on the mass of the body, the distance of the centre of mass of the body from the centre of the earth and the Universal gravitational constant (G). However, the period of swing of a pendulum depends as well on the angular amplitude of displacement of the pendulum bob and this has been proved experimentally (Collins & Bull, 2006). Thus, the approximation of small angular displacement and consequently the assumption that the period of a simple pendulum depends only on the length of the pendulum is an over-approximation. This study has shown that the inclusion of angles of displacement of the pendulum bob in the equation for the period yields a very good value of acceleration of free fall (g) in the laboratory.

II. DERIVATION OF EQUATION OF THE PERIOD OF SWING

In most laboratory manuals used in schools the equation used is:

$$T^2 = 2\pi\sqrt{\frac{\ell}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Equation (1) represents the simple harmonic motion (SHM) which assumes small angular displacements of the pendulum bob during the experiment. The angular approximations being given as

$$\sin\Theta \sim \Theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence the angular dependence of the acceleration of free fall (g) has been eliminated in most calculations of g in laboratory experimental determinations. The scientifically accepted value of acceleration of free fall also called acceleration due to gravity is 9.80665 ms^{-2} (NIST CODATA). This value is often approximated to 9.8 ms^{-2} in most laboratory calculations.

The model for determining g from simple pendulum swings makes the following assumptions: absence of friction due to air, small angular swings, massless inextensible string, and use of a point mass bob. Small angular swing is the most difficult to realise among the assumptions. This is because scientists differ in assigning a value to small angle. Some scientists define small angles as those ranging from 0^0 to 10^0 whereas others consider up to 15^0 as small. Experiences in the laboratory show that students end up displacing the bob of the pendulum through angles that are larger than 10^0 or 15^0 yet they end up with the value of g quite close to 10 ms^{-2}, which is widely accepted as being close enough to 9.8 ms^{-2}, the accepted standard value. In this article, geometry of the simple pendulum has been considered and small angular swing approximation has been eliminated.

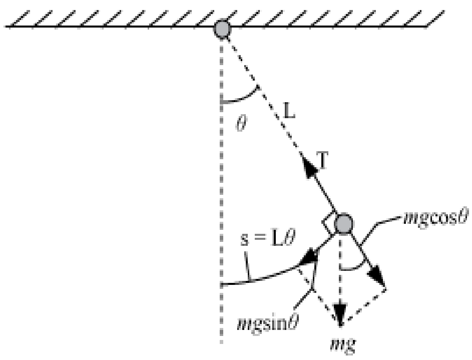

Fig. 1. A free-body diagram of a pendulum bob showing forces on it.

The geometry of Fig. 1 shows that an approximation of small angle as shown in equation (2) allows us to write:

s = LΘ…………………………………………(3)

Thus, without the approximation of small angles we may resort to the following equations and calculations:

The restoring force, F_R, which is tangential to the arc is:

F_R = -mg sin Θ………………………………………(4)

where the minus sign indicates that F_R is in the opposite direction to the displacement of the pendulum bob.

The length of the arc s, in Fig. 1 is given as:

s = L tan Θ = L sin Θ/cos Θ………………………… (5)

But the restoring force may be re-written as

F_R = - ks…………………………………………(6)

where k is the force constant and s is the arc traced by the displaced bob.

So equating (4) and (6) gives:

$-mg\sin\theta = -ks$…………………………………..(7)

Substituting the expression for s from (5) in (7) gives:

mg sin Θ=k L sin Θ/cos Θ…………………………(8)

and

k = mg cos Θ/L ……………………………………(9)

The period, T, of a light spring undergoing a simple harmonic motion is given by:

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g\cos\theta}}$…………………………(10)

when k from (9) is substituted.

Finally, the period, T, of a light body performing a simple harmonic motion has been shown to have angular displacement dependence, provided the length of the pendulum, L, and g are kept constant. Equation (10) shows that a plot of T2 against 1/cosΘ will have a linear dependence. The plot will result in a straight line whose gradient will enable the determination of acceleration of free fall (g), provided the usual laboratory precautions for a pendulum experiment are taken.

III. The Laboratory Experiment

In order to use the derived equation (10) to determine g in the laboratory, the experimental set-up shown in Fig. 2 was arranged and used.

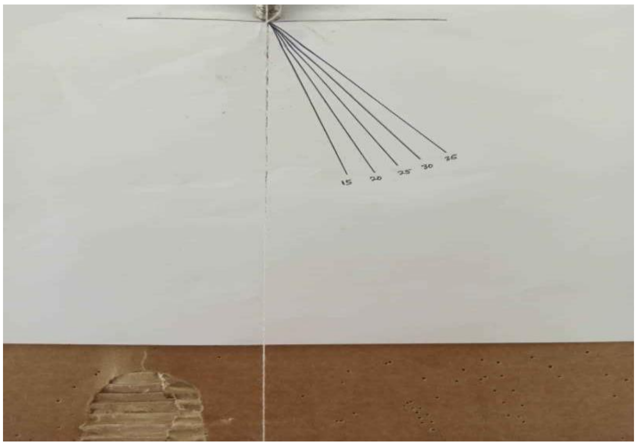

Fig. 2. Set-up for determining g in the laboratory

A bob was attached to a light thread of one metre length and threaded through a split cork in the claws of a retort stand. A cardboard board on which five different angles had been traced out with the aid of a protractor was mounted on the edge of the split cork and behind the pendulum. The pendulum was displaced from the vertical position through an angle, θ, and released to oscillate freely. The period of oscillation taken from the position of maximum displacement was timed with a stopwatch for 20 complete oscillations and tabulated as shown in Table I below.

IV. Results

The data for the angular displacements of the pendulum for 20 swings and subsequent calculations are tabulated in Table I.

A. Precautions

The usual precautions that have been recommended for laboratory experiments for the swinging pendulum were taken in this study to minimise errors.

1. The effects of air resistance were minimised by closing all windows in the laboratory.
2. In timing oscillations, the release of the bob was timed and a complete round was also timed with a stop-watch to determine a complete swing.
3. The string was hung in such a way as to be free from the cardboard on which the angles had been drawn.

B. Sources of Error

Bull (2012) notes that laboratory experiments involving the swinging pendulum suffer errors of design due to two main assumptions: (1) that the string used is inextensible and, (2) that bob swings in a single plane. However, students' experiments in the laboratory are hardly performed in a draught-free environment so the pendulum wobbles slightly and consequently tend to process about the equilibrium position. It is impossible to obtain an inextensible string so it is likely that the weight of the pendulum bob would cause extension in the string. Thus, in this experiment the most likely causes of errors are a) the elasticity of the string, which might have caused slight variation in the length of the string during the experiment and b) since the laboratory was not completely draught-free there might have been precession of the bob around the equilibrium position.

TABLE I: Data for 20 Oscillations of the Swinging Pendulum

Θ^0	1/cos Θ	t_1/s	t_2/s	t_{av}/s	T/s^{-1}	T^2/s^{-2}
5	1.004	37.98	37.95	37.92	1.896	3.595
10	1.015	38.14	38.26	38.20	1.910	3.648
15	1.035	38.58	38.70	38.64	1.932	3.735
20	1.064	39.29	39.15	39.22	1.961	3.844
25	1.103	40.05	39.91	39.98	1.999	3.998

A plot of T^2 versus the 1/cos Θ for data from Table I yielded a linear plot (Fig. 3) whose gradient was calculated for the value of g.

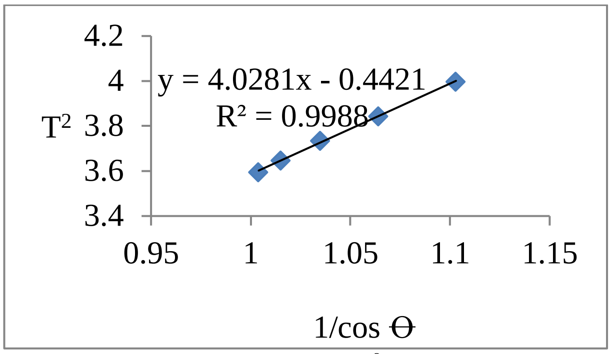

Fig. 3. Plot of period squared (T^2) versus 1/cos (Θ)

In squaring equation (10) one obtains

$$T^2 = 4\pi^2 \frac{L}{g} \cdot \frac{1}{\cos\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

But the equation of the straight resulting from the plot is:

y = 4.0281x - 0.4421............................(12)

Now, comparing equations (11) and (12) one obtains a gradient, m, from which g may be calculated as:

$$m = 4\pi^2 \frac{L}{g}$$

where L = 1 m

$$g = 4\pi^2 \frac{L}{m} = 4\pi^2 \frac{1}{4.0281} = 9.93024 \text{ ms}^{-2} \ldots\ldots\ldots\ldots\ldots (13)$$

and the regression coefficient, R = 0.9994

The value, of g = 9.93024 ms^{-2} which was obtained was only 1.26 % above the accepted value of g = 9.80665 ms^{-2} (Lide, 1993). Also, the regression coefficient, which was 0.9994, showed a deviation of only 0.06 % from complete alignment with the accepted value of g.

V. Conclusion

It is usual that science teachers require of their students to use small angular approximation whereas doing the pendulum experiment. However, the practicality of achieving small angular displacements is almost impossible. Students end up displacing the pendulum bob through angles, which in fact cannot be described as small angles, yet achieve values close to the usual g = 10 ms^{-2} value. This study draws attention to the fact that even bigger angular displacements above 10^0 do yield values of g that can be considered as very good results in the context of students' laboratory experiments.

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