Early Subitizing Development: The Role of Visuospatial Working Memory

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ABSTRACT

The present study explored enumeration processes, serial counting and subitizing, in early childhood. Subitizing is a fast and accurate assessment of small quantities. Early quantitative processing is built off of a “parallel individuation” system, a domain-general, visuospatial system that allows infants to track small sets of objects. This system creates a precise mental model of every object in a small set, similar to subitizing. Subitizing ability helps children to acquire mature symbolic representation of numeral, and deficits in enumeration processes is associate to math disabilities. However, the role of development in enumeration processes and the role of visuospatial working memory is largely unknown. Hence, in the present study, we tested visuospatial working memory’s role in subitizing development in preschoolers. We tested 3 to 5-year-old’s performance in enumeration and visuospatial working memory tasks. Performance in the subitizing range (one to four items) related to individual differences in visuospatial working memory and age, while performance for greater numerocities in the serial counting range did not. Interestingly, controlling for visuospatial working memory span reduced the developmental effect of subitizing, suggesting that both visuospatial working memory and age can affect individual differences in subitizing in young children, which has implications for early childhood education.

Keywords: Development, Serial Counting, Subitizing, Visuospatial Working Memory

I. INTRODUCTION

Two distinct psychological processes support the enumeration of visual stimuli. 1) Serial counting, an effortful and controlled process that is engaged when counting sets of objects greater than four. Reaction time (RT) for serial counting increases linearly; as the number of items increases, RT increases in proportion. 2) Subitizing is a fast and accurate assessment of small quantities (Kaufman & Lord, 1949), typically ranging from one to three items. As opposed to serial counting, RT within the subitizing range barely increases (100 ms per item) as items increase, and therefore, the slope for adults RT remains relatively flat (Akin & Chase, 1978; Mandler & Shebo, 1982; Simon et al., 1998; Trick & Pyllyshyn, 1993).

Similarly, two innate systems support number processing in infants; one for approximate representation of large magnitudes and another precise, automatic system for representation of small quantities (Feigenson et al., 2004) (but see: Leibovich et al., 2015). Infants are born with a precise representation of small amounts of objects (up to four items). In contrast, representations for larger amounts of objects are ratio-dependent, based on Weber’s law. Following this logic, infants’ capacity to track objects one-to-one breaks down for numerocities larger than three (Feigenson et al., 2002). Anobile et al. (2016) added a third system to texture, which is a mechanism for processing a large number of items that cannot be individuated.

While there is general agreement that adults’ subitizing is based on the system for precise representation of small quantities (Feigenson et al., 2002), serial counting requires more skills that are acquired later in development (Feigenson et al., 2004; Starkey & Cooper, 1995). This difference suggests that although subitizing and serial counting are both enumeration processes, they have different underlying skills. Subitizing relies on the capacity to track objects. Serial counting is more complex and requires individualization and localization of objects, switching spatial attention from one object to another, vocally summing the number of objects, and inhibiting the ones already summed. Importantly, serial counting, but not subitizing, involves understanding counting principles (Gallistel & Gelman, 1992) and the development of the verbally mediated symbolic number system (Halberda & Feigenson, 2008). Both serial counting and subitizing are critical, independent components in preschoolers’ numeracy knowledge and serve as central parts in early numeracy development.
assessments (Purpura & Lonigan, 2015). These abilities have been found to predict elementary school mathematics performance (Gray & Reeve, 2014; Hannula-Sormunen, Lehtinen, & Räsänen, 2015).

Carey (2001; 2004) proposed that the symbolic system of quantitative processing is built off of a “parallel individuation” system, a domain-general, visuospatial system that allows infants to track small sets of objects. This system creates a precise mental model of every object in a small set, or other words: subitizing. Then, children learn to represent ordered relations (cardinality principal), in addition to the verbally mediated plus one rule, these developmental trajectories enable the acquisition of the complete set of symbolic numbers (Carey, 2001, 2004). Thus, subitizing is based on infants’ innate ability to track objects, while counting requires additional verbal rules and understanding of cardinality.

Several studies support Carey’s theory that subitizing is based on a visual “parallel individuation” system, instead of the previously suggested analog magnitude system (Piazza et al., 2011; Revkin et al., 2008). Evidence for this comes from adult performance on an estimation task: accuracy and variability of estimations of quantities larger than ten followed Weber’s law, estimation of quantities in the subitizing range (between one to four) did not, which suggests a dissociation between estimation and subitizing (Revkin et al., 2008). Moreover, Piazza et al. (2011) examined the role of visual working memory in subitizing and estimation by applying a dual-task paradigm while participants engaged in subitizing and estimation tasks. The results indicated that subitizing, but not estimation, was negatively impacted from a visual working memory load, suggesting a shared cognitive mechanism. Similarly, Formoso et al., (2017) found a significant relationship between visuospatial working memory (VSWM) and subitizing in 4- and 6-year-old children.

Despite the many studies on early enumeration abilities, there are only a few studies to date that examine the role of VSWM on early enumeration. Our first goal was to map subitizing developmental trajectories in preschool-aged children (3-6-year-olds). The second goal was to examine the relationship of these developmental trajectories with VSWM. Specifically, we examined the development of subitizing and its relationship with VSWM. The children performed a computerized quantity naming task, and a VSWM task. Based on previous research, we expected to find that subitizing range would increase with age. Across participants, we expected to find a relationship between subitizing slope and VSWM span (Formoso et al., 2017), and the developmental effect would mediate this relationship.

II. METHOD

A. Participants

Sixty-two children participated in the experiment. Twelve 3-year-olds, (age range = 33 – 39 months), twelve 4-year-olds (age range = 45 – 51 months) and thirty-nine 5-year-olds (age range = 60 – 72 months). All children were monolingual, native Hebrew speakers with normal vision and no reported learning difficulties. The participants’ parents signed a consent form, and the children received a prize for participation.

B. Tasks

1. VSWM Corsi Block

A computerized version of the Corsi Block test was used to measure the participants’ VSWM span. Nine blue squares (2 cm × 2 cm) were unevenly distributed over a 16 cm × 16 cm quadrant on the computer screen. The positions of the squares were fixed. Each trial began with the presentation of nine squares in blue, then a random sequence of the squares lights up in yellow at a rate of 1 square per second. The squares remained on the screen for 500 ms after the sequence was completed. Then, a black screen was presented for 15 s. Finally, the nine blue squares were presented again, and the participant had to reproduce the sequence in the order in which the squares light up by mouse click (task from the Psychology Experiment Building Language (PEBL); (Piper et al., 2011)).

2. Enumeration

Participants were presented with a random configuration of fish at the center of the computer screen. They were asked to name the number of fish aloud as quickly and accurately as possible. The numbers of fish ranged from one to six and were presented in light green on a black background. Vocal RT’s and responses were input via keypress by the examiner. Each trial started with a fixation asterisk for 300 ms, followed by a blank screen for 500 ms, then the fish array appeared until the child responded, followed by a blank screen while the experimenter recorded the participant’s response. There were 36 trials, each number was presented six times, which were randomized and divided evenly into two blocks.

III. RESULTS

A. RT analysis

We conducted a repeated measurement ANCOVA on the RT’s for the correctly counted fish enumeration trials with age as covariance and the within-subjects factor numerosity (four levels: 1, 2, 3, or 4 fish). Due to low accuracy rates in larger quantities (greater than four), we tested the effect of RT for numerosities with average accuracy rates larger than 50%.

Fig. 1. Reaction times (RT) as a function of numerosity. The result indicated continuity between subitizing and counting. Specifically, RT was longer for 2 items compared to 1 item, and 3 items compared to 2 items and finally, 4 items compared to 3 items.
The main effects for numerosity, $F(3, 180) = 41.89, p < 0.01$, partial $\mu^2 = 0.41$, and age, $F(1, 60) = 19.72, p < 0.01$, partial $\mu^2 = 0.24$, were significant. It took participants more time to respond to two objects compared to one object ($t(61) = -5.15, p < 0.01; M$ difference = 404 ms, $SD = 618$ ms) and to three objects compared to two objects ($t(61) = -4, p < 0.01; M$ difference = 362 ms, $SD = 718$ ms) and to four objects compared to three, ($t(61) = -9.93, p < 0.01; M$ difference = 994 ms, $SD = 789$ ms) (see Fig. 1).

Across numerosities, the correlation between age and RT was negative and significant, $r(62) = -0.50, p < 0.01$. The interaction between age and numerosity reached significance, $F(3, 180) = 20.43, p < 0.05$, partial $\mu^2 = 0.25$ (see Fig. 2).

To understand the interaction, we compared the correlations between RT slopes within the subitizing range (1 to 2 items, 2 to 3 items, and 3 to 4 items) and age, these correlations were significant $r(62) = -0.60, p < 0.001$; specifically, the slope between 1 to 2 items significantly related to age $r(62) = -0.44, p < 0.001$, similarly the slope between 2 to 3 items significantly related to age $r(62) = -0.44, p < 0.001$ while the slope between 3 to 4 items did not significantly $r(62) = -0.18, p = 0.16$ (see Fig. 2).

Moreover, we calculated individual subitizing range by mapping standardized RT for one to four items and combining bilinear fit and sigmoid fit (Leibovich-Raveh et al., 2018). The RT data were fit first with a sigmoid function as with the basic sigmoid method using a non-linear error minimization algorithm. Then two lines were fit to the sigmoid curve; one with a slope of zero and an intercept where the sigmoid curve crosses the y-axis at x = 0 (i.e., the subitizing line), and a second line which is equivalent to the tangent at the inflection point (Leibovich-Raveh et al., 2018). Fig. 3. represents the individual range by age groups. As can be seen in Fig. 3., the subitizing range for the youngest participants (32-38 months old) was on average 0 - 1.5 items, for slightly older children (39-45 months old) it reached 1.75 items, and for the oldest participants (48 months and older) the range went beyond two items.

Fig. 2. Correlation between reaction times (RT) slope (between 1 and 4 items) and age. The result indicated a negative and meaningful correlation between age and RT, as participant age increased RT decreased.

B. Entering VSWM Span as a Covariate RT

To explore the relationship between VSWM and subitizing, we conducted the same ANCOVA as above and added VSWM span as a covariate. The main effect of numerosity $F(3, 177) = 42.81, p < 0.01$, partial $\mu^2 = 0.42$, and age, $F(1, 59) = 4.55, p < 0.05$, partial $\mu^2 = 0.072$, remained significant. Additionally, the interaction between age and numerosity, $F(3, 177) = 7.15, p < 0.01$, partial $\mu^2 = 0.17$, remained significant. Importantly the main effect of VSWM span and the interaction between VSWM span and numerosity were significant, $F(1, 59) = 8.19, p < 0.01$, partial $\mu^2 = 0.13$, and $F(3, 177) = 4.3, p < 0.01$, partial $\mu^2 = 0.07$, respectively. VSWM span negatively associated with the slope between one to two items ($r(62) = -0.40, p = 0.001$) and the slope between two to three items ($r(62) = -0.33, p < 0.01$), and the slope between three to four items ($r(62) = -0.28, p < 0.05$). Additionally, VSWM span interacted with the slope of one to four items ($r(62) = -0.58, p < 0.001$) (see Fig. 4.).

Fig. 3. Individual subitizing range by age. The age range is marked on the title of the graphs. The red sigmoid curve is fit to the normalized RT data. A tangent line is then fit to the inflection point of the curve and a subitizing line of slope zero intersecting the y-axis at the same point as the sigmoid curve. The intersection point of these two lines defines the upper bound of the subitizing range. On the y axis appear normalized RTs and on the x axis appear number of presented items between 1 to 4.
Across age groups, children with flatter subitizing slope had higher VSWM span. To further explore the relation between VSWM span, age, and subitizing slope (1 to 4) items we examined the relationship between age and subitizing slope with VSWM as a mediator using PROCESS macro. The results indicated that age had both a direct effect on subitizing slope (effect = -13.5, SE = 4; t = -3.37, p = .013, LLCI = -21.5, ULCI = -5.4) and an indirect effect on subitizing slope (effect = -6.88, SE = 2.5; LLCI = -12.6, ULCI = -2.6) (see Fig. 5).

To understand the interaction, we calculated the subitizing slope for accuracy rates and compared the correlations between these slopes (1 to 2 items, 2 to 3 items, and so on) with age. Age significantly correlated with accuracy of numerosities within the subitizing range, $r$($62$) = 0.60, $p < 0.001$, but not with accuracy for numerosities in the counting range, $r$($62$) = 0.20, $p = 0.11$. Specifically, the slope between 1 to 2 items, 2 to 3 items, and 3 to 4 items all significantly related to age ($r$($62$) = 0.33, $p < 0.05$; $r$($62$) = 0.29, $p < 0.05$; $r$($62$) = 0.28, $p < 0.05$, respectively) (see Fig. 7.).

Across numerosities, the correlation between age and accuracy was positive and significant, $r$($62$) = 0.60, $p < 0.01$. The interaction between age and numerosity reached significance, $F$($3, 180$) = 14.06, $p < 0.01$, partial $\mu^2$ = 0.16 (see Fig. 6.).

C. Accuracy Analysis

We conducted a repeated measurement ANCOVA on the accuracy rates for numerosity as a within subject factor (1-6 items) and age as a covariate.

The main effects for numerosity, $F$(5, $300$) = 23.24, $p < 0.01$, partial $\mu^2$ = 0.28, and age, $F$(1, $60$) = 34.17, $p < 0.01$, partial $\mu^2$ = 0.36, were significant. Accuracy rates were lower for three items compared to two items ($t$(61) = 2.15 , $p < 0.05$; $M_{\text{difference}}$ = 0.048, $SD$ = 0.18), for four items compared to three items ($t$(61) = 3.08 , $p < 0.01$; $M_{\text{difference}}$ = 0.082, $SD$ = 0.21) and for four items compared to five items ($t$(61) = 3.23 , $p < 0.01$; $M_{\text{difference}}$ = 0.079, $SD$ = 0.19) (see Fig. 5.).

Across numerosities, the correlation between age and accuracy was positive and significant, $r$($62$) = 0.60, $p < 0.01$. The interaction between age and numerosity reached significance, $F$(3, $180$) = 14.06, $p < 0.01$, partial $\mu^2$ = 0.16 (see Fig. 6.).

D. Entering VSWM Span as a Covariate Accuracy

To explore the relationship between VSWM and subitizing, we conducted the same ANCOVA as above and added VSWM span as a covariate. Entering memory span as a covariate did not change the reported effect and yielded marginally significant effects of VSWM and the interaction between VSWM and numerosity.
IV. DISCUSSION

A. Summary of the Main Results.

1) The interaction between age and subitizing was significant, indicating that subitizing slope is modulated by age in preschool-aged children.
2) Subitizing range was very narrow (1.5 items) among the youngest children (32-38 months) in the current study and increased to more than 2.25 items among the older children (66-72 months).
3) The developmental effect of the subitizing range was reduced when we entered VSWM as a covariate.
4) VSWM modulated the relation between age and subitizing slope (directly and indirectly).
5) Performance in the subitizing range (one to four items) related to individual differences in VSWM span and age, while performance for greater numerosities in the serial counting range did not.

B. Developmental Trajectories of Subitizing.

Subitizing is an innate ability to process small quantities. A previous study analyzed 2- to 5-year-old’s performance in a non-symbolic comparison task and found that children’s accuracy was above chance only when comparing small quantities (Starkey & Cooper, 1995). They also mapped a developmental trajectory for subitizing. For 2-year-olds, the subitizing range included one to three items, for 3.5-year-olds, the range increased to four items, and for 4-year-olds, it increased to five items (Starkey & Cooper, 1995). Most studies with adults report a subitizing range of one to three (Dehaene & Cohen, 1994; Landerl et al., 2004; Mandler & Shebo, 1982) or one to four items (Piazza et al., 2002; Trick & Pylyshyn, 1993). However, similar to the present study, previous studies with adult participants used a quantity naming task and not the quantity comparison task Starkey & Cooper (1995) used when examining subitizing in young children. A previous study that used a quantity naming task found that 4-year-olds have a similar subitizing range to adults (Jansen et al., 2014). The subitizing range was defined by the discontinuity between RT slopes of the subitizing and counting range.

We found continuity in the RT slope across the subitizing and counting range (a significant increase in the RT was found between 1 to 2 objects, 2 to 3 objects, and 3 to 4 objects) is contradictory to Jansen et al. (2014). Moreover, our younger participants (32 – 45 months) had a subitizing range of fewer than two objects. Please note that our group was younger than Jansen et al. (2014), suggesting a developmental effect of subitizing in the ages of three to six.

Additionally, our results seem to contradict Starkey & Cooper (1995), which found subitizing among 2-year-olds; we found no evidence of subitizing among our younger participants. This difference can stem from the different tasks used and the related task demands. The quantity comparison task requires only representation of quantities; therefore, it is slightly easier for younger children, while the quantity naming task, beyond quantity representation, requires mapping of quantities to their verbal tags. Even though the parallel individuation system of representation of small quantities is evident as early as 2-years-old (Carey, 2001, 2004), the fully automated connection between the verbal tag and small quantities is not fully developed in our younger participants.

C. The Role of VSWM in Subitizing

Current approaches to subitizing claim that it is based on a domain-general, visuospatial, precise object tracking system, rather than quantity representation (Piazza et al., 2011; Revkin et al., 2008). To understand the relationship between VSWM and subitizing, Piazza et al. (2011) conducted a study in which adults performed a VSWM task and a subitizing task. They found a positive and significant correlation between subitizing range and VSWM span. Formoso et al. (2017) found a significant and moderate relation between subitizing slope and VSWM span in children (4- and 6-years-old). Here, we found a similar correlation between VSWM and subitizing among preschool-aged children. Importantly, we found that VSWM modulated the effect of development on the subitizing range. Please note that while Piazza et al. (2011) used a visual working memory task (memory for object colors), we and Formoso et al. (2017) used a spatial working memory task (memory for object locations). Altogether, these results suggest that subitizing is based on object individuation capacity related to both its location and identity.

The current study was the first that examined the relationship between subitizing and VSWM in very young children (from 3 years old). As predicted, we found a developmental effect of subitizing. However, we found a developmental effect for VSWM as well. After we entered VSWM span as a covariate to the quantity naming task, the significant effect of age and the interaction between numerosity and age group were reduced. Hence, the developmental effect of subitizing between three to six years of age was partially mediated by the effect of VSWM. This result highlights the importance of individual differences in VSWM as an explanatory factor in subitizing abilities over developmental factors.

The present study’s results partially confirmed Carey’s theory on early quantitative understanding (2001; 2004). We found evidence for the importance of the parallel individuation system in the representation of small quantities in the critical role of VSWM in subitizing. Specifically, entering VSWM span as a covariate reduced the effect of age. Moreover, this result presents the specific connection between subitizing slope and individual VSWM abilities as an initial step in symbolic representation development.

Our finding regarding VSWM is particularly interesting due to recent research that found visuospatial WM as a significant predictor of math ability for both typically developing children and children with developmental dyscalculia (Ashkenazi et al., 2013; Szücs, 2016; Szucs et al., 2013). These studies were based on the performance of children and adults. The current study results suggest that the relationship between mathematics and VSWM exists even in early childhood and can be applied in preschool classrooms. Pre-screening for children with possible future math difficulties can be conducted in preschool by assessing enumeration and VSWM skills.

V. CONCLUSION AND LIMITATIONS

Due to the young age of our participants, recruitment was a challenge, as such the study had a small group of participants, especially in our youngest age groups. Accordingly, it is important to replicate the innovative results in a larger sample.
The present study examined enumeration processes in a group of preschool children and discovered that the period between 3- and 6-years of age is critical for the development of subitizing. Unlike the common findings among adults, we found continuity in enumeration performance for numerosities across the subitizing and serial counting range. However, the subitizing range was modulated by age. VSTM related to subitizing and reduced the age effect in subitizing. The present study provides additional evidence that subitizing and serial counting are distinct processes developmentally (Feigenson et al., 2004; Gallistel & Gelman, 1992; Piazza et al., 2002) as well as a significant relationship between subitizing and VSTM (Piazza et al., 2010; Piazza et al., 2011).

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CONFLICT OF INTEREST

No competing financial interests existed. The authors declare no conflict of interest.

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