Problem-Solving among Math Anxious Individuals: The Role of Advanced Strategy and Testing of Online Anxiety

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ABSTRACT

Math anxiety (MA) is negatively associated with math performance. To explore the role of online or stable MA in advanced strategy use, 52 college students solved verbal multiple-choice mathematical problems. MA was negatively related to advanced strategy use. Online anxiety and advanced strategy use were unique predictors of accuracy. Using mediation analysis, we discovered that advanced strategy uses partially mediated the relation between MA and math performance. The results show that the selection of a non-advanced strategy is one of the cognitive processes that explained the negative relation between MA and performance, and that MA has a strong contextual component.

Keywords: Math Anxiety, Mathematical Problem, Online Math Anxiety, Problem-Solving, Strategy Selection.

I. INTRODUCTION

Math anxiety (MA) is a feeling of tension and fear that interferes with one’s ability to manipulate numbers and solve mathematical problems in everyday life and in academic situations (Suinn et al., 1972). People with MA tend to avoid math situations, which govern their lifestyle decisions and result in everyday life implications, such as college major and occupational choices (Georges et al., 2016; Hembree, 1990).

There is an ongoing debate in relation to the origin of MA, and the situational influences of MA in the case high anxiety (Ashcraft, 2002; Ashcraft & Kirk, 2001). Ashcraft and colleagues showed that while HMA individuals do not exhibit difficulties in basic addition and multiplication in whole-number arithmetic problems, there is evidence that they find difficulty with more complicated addition problems, such as those requiring a carry-over operation (e.g., 37 + 96, or 5 + 8). These results led them to assume that HMA individuals have difficulties, especially in complex arithmetical operations that require working memory (WM). Hence, HMA individuals have limited WM resources available while solving math problems, due to anxiety-related ruminations, thus resulting in poor math performance (Ashcraft, 2002; Ashcraft & Kirk, 2001).

Newer theories regarding MA (Maloney, Risko, Ansari, & Fugelsang, 2010), show that MA is accompanied by difficulties in lower numerical abilities, with minimal WM demands. Specifically, HMA individuals had difficulties in the enumeration in the counting range, but not within the subitizing range. Later, in the same group (Maloney et al., 2011) tested number comparison in a group of participants with high or low MA. Results indicated that HMA individuals had a steeper numerical distance effect (NDE) than LMA individuals, which presumably resulted from less accurate representations of numerical magnitude (see Dietrich et al., 2015 for an alternative explanation). Combining the results of the two experiments, Maloney and colleagues concluded that MA does not only affect complex and high-level mathematics, but rather is also accompanied by difficulties in basic numerical processing. One way to understand the origin of MA and MA’s situational influences in a case of high anxiety is to look at strategies used during the solution of math problems.

A. Math Anxiety and Strategy Selection

Every problem can be solved using multiple strategies. The chosen strategy may depend on our age, cognitive abilities, experience, feelings or thoughts, and time limit. The selection of strategy becomes adaptive as we gain experience, and we tend to choose the efficient strategy leading to the fastest and most accurate solution (Siegler, 1999). This means that a more advanced strategy, i.e., appearing later in development, is also a more efficient strategy in terms of either accuracy, reaction time (RT), or both (Lemaire & Siegler, 1995; Siegler, 1999). With training and development, children learn adaptively to choose a mixture of strategies related to problem characteristics (Lemaire & Siegler, 1995; Siegler, 1999).

Anxiety reduces attentional control and impairs executive functions. Specifically, it impairs inhibition of dominant responses and shifting between tasks (Eysenck et al., 2007). These abilities, among others, have an important role in the selection of advanced and efficient strategies (Miyake et al.,
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Another & Wolter, questionnaire Anxiety previous (2018) the groups more was advanced strategies. Similarly, Ramirez et al. (2016) tested strategy to use in children in relation to MA and WM individual abilities. The relation between MA and maths achievements was modulated using advanced memory-based strategies. Note that high, more than low, WM individuals, tend to use more advanced memory-based strategies. Hence, their math performances are more vulnerable to being affected by MA. Later, Si and colleagues showed that MA affected the choice of strategies for computational estimation. HMA and LMA groups were presented with an additional exercise and asked to estimate the sum. HMA individuals were less accurate in their estimation, i.e., they used a less appropriate strategy for the solution (Si, et al., 2016). Last, Ashkenazi and Najjar, (2018) examined the solution of simple non-carry addition problems (e.g., 54 + 63) and complex carryover addition problems (e.g., 59 + 63). The results indicated that high MA participants showed difficulty in the harder carry condition. Testing the strategy selection mechanism among high MA participants, they found that in the carry condition:

1) the common strategy was used less often compared to low MA participants
2) high MA participants employed unusual strategies more often than low MA participants

B. Online MA Testing Versus Traditional Self-Report Questionnaires of State MA

A current problem with MA diagnostic criteria is that most previous studies with adults used self-report questionnaires with hypothetical situations, such as the Mathematics Anxiety Rating Scale (MARS), or shorter versions of this questionnaire (AMAS; sMARS; MARS-R). These questionnaires test mostly state MA while ignoring online testing in the math anxiety (Bieg et al., 2014; Bieg, Goetz, Wolter, & Hall, 2015; Buehler & McFarland, 2001; Wilson & Gilbert, 2005). However,

1) few current studies have clearly indicated a discrepancy between state and trait MA. For example, one study that tested students in algebraic problem solving under a time limit and no time limit, discovered complex relations between contextual factors and MA level (Trezise & Reeve, 2018). Another study that compared trait and state anxiety in children found a relation between math achievement and MA for state anxiety but not trait anxiety (Orbach et al., 2019).

2) Multiple theories link context to anxiety. For example, the “choking under pressure theory” in math indicates that under low pressure, students use math strategies based upon high working memory demand, whereas under high pressure they use low-cognitive demand strategies, demonstrating the role of problem complexity in anxiety arousal (Beilock & Carr, 2005; Beilock & De Caro, 2007). Hence, it is important to test the anxiety arousal link to specific problems.

C. The Current Study

Two main aims guided the present study. The first aim was to test the relations between MA, the advanced strategy uses, and math performance. High MA individuals use the same solution strategy repetitively and do not change adaptively (Si et al., 2016). Hence, we expected to find that participants with high MA would show lower use of advanced strategies. Moreover, we assumed that both MA level and advanced strategy use would be unique predictors of math performance. Finally, we believe that the relation between MA and maths performance will be mediated by advanced strategy use.

The second goal of the present study is to compare online MA testing with traditional offline MA testing. We will examine MA once with the traditional approach using a hypothetical questionnaire (MARS-S), and participants will rate their anxiety level once after solving a mathematical problem (online MA). We expected that online MA testing will be a better predictor of performance than a traditional MA questionnaire.

In the present experiment, participants will be presented with verbal multiple-choice mathematical problems. A main feature of the problems was that there was no algorithm or rule of thumb to solve them, but there were rather multiple ways to arrive at the solution (see method section). To evaluate strategy use, each participant first selected the answer to the problem and then reported his solution strategy. Next, to evaluate online anxiety, every problem was followed by a request to rate the anxiety level for that problem, on a scale of 1 (not at all) to 9 (very anxious) (see method section for further details). Prior to the experiment, we conducted a pilot study that classified specific solution strategies for the experimental problems as advanced or not advanced. Based on the pilot study results, we classified solution strategies (see method section for further details).

II. Method

A. Participants

Fifty-two students participated in the study (85% women), gave written informed consent, and received either course credits or 30 New Israeli Shekels (equal to about 8.55) for their participation. This study was approved by the Ethics Review Panel (ERP) of the Hebrew University of Jerusalem, Israel. All participants were university students. Approximately half were from the Department of Education at the Hebrew University of Jerusalem and received course credit or 30 NIS for the experiment (mean age = 27.7, S.D = 6.78), and approximately half were from Teachers’ College and received only course credit for participation (mean age = 29.5, S.D = 6.57). All participants were native Hebrew speakers, with no mathematical learning disabilities. They were invited to participate in a study about math problem solutions.

Three participants were excluded from the sample. One participant asked to leave, another reported having Dyscalculia, and the last one reported he had no basic mathematical knowledge. More women than men participated in the study (44 women, 8 men).
B. Procedure

The experiment was computerized and took place in a lab at the Mount Scopus Campus, the Hebrew University of Jerusalem, or in the computer room of the Givat Washington College. Prior to the experiment, participants signed a consent form and filled out a demographic questionnaire. After completing the Hebrew Mathematics Anxiety Rating Scale - Short Version (MARS-SV), participants were tested on a Mathematical Problem-Solving task. The questions in the Mathematical Problem-Solving task were presented in random order, as will be explained below.

C. Tasks

1) Mathematics Anxiety Rating Scale–Short Version (MARS-SV)

Participants answered a Hebrew version of the Mathematics Anxiety Rating Scale–Short Version (MARS-SV; Richardson & Suinn, 1971). The value of Cronbach’s Alpha for the MARS-SV Hebrew version across items was high = 0.953. The questionnaire included 30 items describing academic and routine situations that may arouse anxiety in individuals, and participants were asked to rate how anxious they would feel in these situations, between 1 (not at all anxious) to 5 (very anxious). Math Anxiety score for each participant was calculated as the sum of the 30 items’ scores, which ranged between 30 and 150. The MARS 30-item was composed of items derived from three-factor analytic studies of the MARS 98-item.

2) Mathematical Problem Solving

16 multiple-choice questions were presented on a screen, and the participant was asked to choose the correct answer. The problems were chosen from a question repository of 5th-6th grade levels (all problems were collected from the A+ Click website (https://www.aplusclick.org/) and were translated into Hebrew by the authors of the paper, after receiving permission to use the problems). Each problem was followed by a request to explain out loud in detail how the participant solved the problem, and finally, the participant was asked to rate how anxious did s/he feel while solving the problem, on a scale of 1 (not at all) to 9 (very much). The experimenter recorded the participants’ answers. Measures of accuracy and RT were automatically recorded.

Half of the problems were with graphical demonstration and a half without (see Appendix A and Appendix B for the specific problems). The problems were chosen based on a pilot study run by the authors prior to the experiment. The problems were numerical in nature and included verbal mathematical problems. 32 multiple-choice problems were chosen from the A+ Click website. In the pilot study, 44 participants were asked to solve the problems, explain how they solved each problem, and rate its difficulty level on a scale of 1 (very easy) to 10 (impossible). The problems were presented in pseudo-random order that was set beforehand. 16 problems that met the following criteria were chosen for the study: (a) had at least 2-3 different strategies for the solution, and (b) 60-90% of the participants succeeded in solving them. Appendix C summarizes the results of the pilot study. In addition to choosing the problems for the experiment, we used the pilot’s results to define the strategies for each question and classify them in terms of efficiency and advancement.

3) Coding the Math Problem Solving Task

In addition to measures of accuracy and reaction time, we investigated the strategies used to solve the problems with or without graphical demonstration. Based on the pilot study, two independent experts classified the strategies into advanced or not advanced for every question, relying on a developmental acquisition of strategies, as well as on the efficiency of strategies (Lemaire & Siegler, 1995). For example, counting using fingers or counting each square are less advanced strategies in comparison to spatial visualization of the problem or calculating area by using a formula. The classification of the possible strategies was used as a set of guidelines when coding the data of the current study (see appendix 1 for classification of strategies into advanced and non-advanced for each of the problems).

III. RESULTS

A. Descriptive statistics

Table I summarizes the descriptive statistics in addition to the correlations between variables

![Fig. 1: Relations between MA offline scores (tested by MARS-SV) and online anxiety rate.](image1)

![Fig. 2: Relations between MA offline scores (tested by MARS-SV scores) (A) or online MA rate (B) and accuracy rates.](image2)
First, as expected, we found a low-to-medium positive correlation between the offline MA (tested by the MARS-SV) score and the online MA rating: \( r(50) = 0.36, p = 0.008 \) (see Fig. 1).

Both offline and online MA correlated negatively to accuracy, \( r(50) = -0.29, p = 0.02 \); \( r(50) = -0.63, p = 0.005 \), for offline and online MA respectively. However, the correlation between online MA and accuracy was stronger than the correlation between offline MA and accuracy, \( Z = 2.55, p = 0.005 \) (see Fig. 2). Only online MA \( r(50) = -0.48, p = 0.005 \), but not offline MA \( r(50) = -0.20, \text{n.s.} \), was correlated to advanced strategy use (see Fig. 3). Hence, the correlation between online MA and advanced strategy use was stronger than the correlation between offline MA and advanced strategy use \( Z = 1.9, p = 0.023 \).

B. Regression Analysis

We ran hierarchical regression analyses to predict the accuracy rate (representing math performance). In the first block, we included scores from MARS-SV (online MA). In the second block, we added advanced strategy use. In the third block, we added online MA as a predictor. The entire model reached significance from the first block \( (R^2 = 0.08, \ p < 0.05) \). The effect of MARS-SV score was highly predictive \( (\beta = -0.29, \ t(50) = -2.1, \ p < 0.01) \). Higher offline MA predicted lower accuracy. The next block add significant explained variability \( (R^2 \text{ change} = 0.35, \ p < 0.01) \). The effect of strategy use was significant \( (\beta = 0.61, \ t(50) = 5.44, \ p < 0.001) \), specifically, more advanced strategy predicted higher accuracy. At that point, the effect of the MARS-SV score was no longer significant \( (\beta = -0.17, \ t(50) = -1.5, \ p = 0.14) \).

The last step entered online MA and added significant explained variability \( (R^2 \text{ change} = 0.01, \ p < 0.01) \). The effect of the online MA score was highly predictive \( (\beta = -0.29, \ t(50) = -2.1, \ p < 0.01) \). Higher MA predicted lower accuracy \( (\beta = -0.29, \ t(50) = -2.1, \ p < 0.01) \). At that point, the effect of strategy remained significant, and the effect of offline MA remained non-significant (see Table II).

C. Mediation Analysis

To further explore the relation between MA and math performance (accuracy) and advanced strategy use (as a mediator), we examined this relationship using PROCESS macro (model 3). Model 3 looks at a mediation effect of advanced strategy use on the relation between MA and math performance. Following the correlation analysis and the results of the regression analysis, we used only online anxiety (but not MARS-S) in the model, as the independent variable.

The results indicated that online MA has both a direct effect on math performance \( \text{effect} = -0.42, \ SE = 0.11; \ t = -3.81, \ p = 0.0004, \text{LLCI} = -0.64, \text{ULCI} = -0.2 \) and an indirect effect by advanced strategy use \( \text{effect} = -0.21, \ SE = 0.07; \text{BootLLCI} = -0.34, \text{ULCI} = -0.082 \) (see Fig. 4).
A few fundamental questions guided the present study:

1) What is the role of advanced strategy use in MA? Previous studies with MA have usually examined strategy selection during simple operations such as 23 + 45 or 2 × 3 with an algorithm or rule of thumb to solve them (Imbo & Vandierendonck, 2007; Ramirez et al., 2016; Si et al., 2016; Ashkenazi & Najjar, 2018). However, it has been suggested that MA affects more complex problem solutions than simple ones (Ashcraft, 2002; Ashcraft & Kirk, 2001). Hence, here we used verbal multiple-choice mathematical problems. The main feature of these problems was that there were various ways to arrive at the solution. To ensure problem difficulties and several solution strategies, we used a pilot study to select specific problems and classified solution strategies as advanced or non-advanced. This procedure enabled us to determine the role of advanced strategies usage in students with MA, and in the relations between MA and maths performance. MA was negatively related to advanced strategy use. Hierarchical regression analysis found that online anxiety and advanced strategy use were unique predictors of accuracy. Using mediation analysis, we found that advanced strategy use partially mediates the relation between MA and maths performance. Accordingly, a direct path between online MA and accuracy was also observed.

2) What are the discrepancies between traditional definitions of MA with self-report questionnaires, and online testing of MA?

Several theories point to the situational effect of MA, hence looking at online anxiety should clarify the relation between math performance and MA. In line with this view, all the findings indicated that online MA is a better predictor than traditional MA testing. First, both online MA and offline MA were negatively correlated with math performance. However, that correlation was stronger in online MA than in offline MA. Second, online but not offline MA was negatively correlated with advanced strategy use. Last, hierarchical regression analysis found that when both offline and online MA were entered to predict math performance, online MA was a significant predictor of math performance, whereas offline MA was no longer a significant predictor.

A. Participants with high MA use less advanced strategies than participants with low MA

Multiple studies have considered how stress influences strategy selection, whether stressors in mathematics drive the selection of more efficient strategies, or whether they cause suboptimal strategy selection by inducing interfering worrying thoughts (for review, see Caviola et al., 2017). The current results hint that stressors in mathematics drive the selection of suboptimal strategies. One potential mediator that links stress to strategy selection is WM. Ashcraft and Kirk (2001) and Ashcraft and Krause (2007) found that WM is negatively affected by the interference of MA. Another potential mediator that links stress to strategy selection is attentional control. The attentional control theory suggests that anxiety in MA disrupts participants’ ability to control attention, making them more susceptible to distraction (Eysenck & Calvo, 1992; Eysenck & Derakshan, 2011). Note that according to the WM theory of MA and the attentional control theory, anxiety should reduce performance in complex more than simple arithmetic problems. Hence, due to reduced WM availability or low attentional control, high MA participants in the current study showed lower usage of advanced strategies in complex verbal problems.

Another aspect that was observed in the current study is the relationship between performance and anxiety. Previous studies as well as the current study found a negative correlation between MA and maths performance (Ashcraft, 2002). The current study partially uncovered the cognitive mechanism underlying this connection. High MA tends to resolve in the usage of non-advanced strategies and thus results in decreased math performance. Note, however, that in the current study, a direct path was also found between MA and maths performance, indicating that additional cognitive mechanisms, beyond strategy selection, may explain the relations between MA and maths performance. For example, it has already been suggested that individuals who suffer from high levels of MA tend to avoid math, have negative emotions towards it, and show lower achievements (Ashcraft, 2002; Carey et al., 2016; Hembree, 1990).

B. Online MA is a better predictor of math performance than offline MA

Most previous studies that tested MA levels using self-report questionnaires detected an offline measure rather than testing anxiety levels while solving math problems (Bieg et al., 2014; Bieg et al., 2015; Buehler & McFarland, 2001; Wilson & Gilbert, 2005). MA offline testing is based upon the implicit assumption that MA is a stable condition, and not influenced by math problem type. However, “the choking under pressure theory” in math indicates that complex problems in math can themselves act as a stressor and arouse anxiety (Beilock & Carr, 2005; Beilock & DeCaro, 2007). Accordingly, using a unitary offline measurement of MA ignores the contextual effects of anxiety. Testing online MA allows observation of the mixed effect of the MA state and the contextual effect of a specific problem. Hence, creating a highly sensitive measurement of MA.

**IV. DISCUSSION**

**CONCLUSION**

In the current study, we tested how strategy selection is affected by MA level. Additionally, the present study examined the role of online MA testing, in addition to the more traditional self-report questionnaire of MA.

First and foremost, our results demonstrate that high MA participants use advanced strategies less often than low MA participants. Moreover, we found that MA and advanced strategy selection both affect math performance. Finally,
advanced strategy selection partially mediates the relation between MA and maths performance, revealing the cognitive mechanism underlying anxiety ability relations.

Finally, our results clearly demonstrate that testing online anxiety provides greater value than using a more traditional approach to MA, by demonstrating the role of context in MA.

**Appendix**

**Appendix A. Examples of Problems with Graphical Demonstration**

<table>
<thead>
<tr>
<th>Problem and answers</th>
<th>Drawing</th>
<th>Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many small circles are not inside all three squares?</td>
<td><img src="image1" alt="Drawing" /></td>
<td>Advanced: Calculate all the circles by multiplication, and subtract the circles that are common to all the squares. Not advanced: Manually count all the circles that are not inside all three squares.</td>
</tr>
<tr>
<td>1. 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. 19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| There are 5 coins in the first pile, 8 in the second, 11 in the third and 16 in the forth. What is the least number of coins that I have to move to make these 4 piles have the same number of coins? | ![Drawing](image2) | Advanced: Average strategy. Calculation of the total amount of coins and division to number of piles. Not advanced: Manually moving coins between piles. Trial and error. |
| 1. 6 | | |
| 2. 7 | | |
| 3. 8 | | |
| 4. 5 | | |

| Mr. Cohen has a large piece of land, and he divided it into 4 parts as pictured here. Which two parts have the same area? | ![Drawing](image3) | Advanced: Spatial manipulation. Comparison of the areas and visually manipulating squares. Not advanced: Manually count the squares and compare the areas. |
| 1. A & B | | |
| 2. A & C | | |
| 3. C & D | | |
| 4. B & C | | |

**Appendix B. Examples for Problems without Graphical Demonstration**

<table>
<thead>
<tr>
<th>Problem and answers</th>
<th>Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which one of the possibilities equals 555?</td>
<td>Advanced: Eliminating answers, without calculating the full solution. Not advanced: Converting into numbers and calculating the answers.</td>
</tr>
<tr>
<td>1. 5 hundred, 15 tens, 5 units</td>
<td></td>
</tr>
<tr>
<td>2. 4 hundred, 15 tens, 5 units</td>
<td></td>
</tr>
<tr>
<td>3. 4 hundred, 4 tens, 10 units</td>
<td></td>
</tr>
<tr>
<td>4. 2 hundred, 4 tens, 10 units</td>
<td></td>
</tr>
</tbody>
</table>

| The sum of all odd numbers from 0 to 10 is: | Advanced: Using the even principle. Sum of odd number of odd numbers is an odd number. Not advanced: Summing all the numbers for which the condition holds. |
| 1. An even number | | |
| 2. An odd number | | |
| 3. Equal to 0 | | |

| Haim was on a diet, and he reduced half his weight. Before the diet, he weighed 189 kg. What is his weight after the diet? | Advanced: Decomposition of the equation. Not advanced: Straightforward calculation of the equation. |
| 1. 95.5 | | |
| 2. 90.5 | | |
| 3. 94.5 | | |
| 4. 90.5 | | |
CONFlict of interest

The authors declare that they do not have any conflict of interest.

References


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