Why Mathematics at the High School Level Evolved to Include Linguistics

Kwame Yankson

ABSTRACT

This article foregrounds the fundamental role that language can play in communicating mathematical ideas and contexts. The findings from this article contribute to an understanding of different ways textbook series with particular orientations make opportunities available for students to develop forms of agency and autonomy during classroom learning. The article also contributes methodology for analyzing mathematical texts of different genres.

Keywords: cognitive analysis, linguistic analysis, problem solving, curriculum.

I. INTRODUCTION

The present work is inspired by the idea that mathematics as a social and as a learning activity, although it can, does not always and only occur as a collection of related mathematical symbols. That words, and language, do and should indeed play a central role in the communication of mathematical ideas such as conjectures, proofs, and theorems. This article is premised on the notion that all mathematical ideas can and should be expressible only as words. Words, and language, therefore, play a fundamental role in the intellectual and social transactions of mathematics. This position is already being taken for granted among mathematics professionals. Mathematics journals, monographs, and textbooks are all written and communicated in a formal language peculiar in disciple-specific ways. Language use is particularly powerful in affording researchers and learners alike the ability to express choice, and hence human agency, in their intellectual actions and interactions (Pickering, 1995). It is thus a boon to the worlds of scientific inquiry, social and arguably even natural, that there are tools available today (Halliday & Matthiessen, 2014) that empower researchers and teachers of mathematics who choose to study in deliberate ways the language used in mathematics, be it in research or in learning (Morgan, 2016). This article presents insights from the study of two textbook series that incorporate different philosophies of problem-based learning (Hmelo-Silver, 2004) in the lesson texts and mathematical tasks that serve as the primary instructional medium in each textbook series.

II. LITERATURE REVIEW

Problem-based learning (Hmelo-Silver, 2004), hereafter, PBL, is an instructional orientation wherein the primary medium of teaching and learning is via guiding learners in their work on different problems which inform both theory and practice. In the context of mathematics, PBL involves solving problems of different kinds as a way of learning mathematics, which can be studied quantitatively and qualitatively (Miles & Huberman, 1994). The mathematician George Pólya who galvanized the importance of problem solving in mathematics education in his promotion of heuristics gave a four-step approach to guide students when solving challenging problems for which there was no immediate method (Pólya, 1971). In Pólya’s approach, students first need to understand the problem they are attempting to solve. To understand the problem, they must ask themselves a number of general questions that can serve as a plan to scaffold students toward a solution. The questions students need to ask themselves concern what the unknowns are, what the data or the givens are and what the conditions are for those data. Once students understand what the problem requires, they can then put together a plan that serves as a method for the problem. This article reviews literature pertaining to textbooks and curriculum materials, to opportunities to learn, to agency and autonomy, and to problem solving.

It is well-known in mathematics education that mathematics problems, which can also be called mathematical tasks, come in different forms, and are also used in different capacities in teaching and learning (Hsu & Silver, 2014) and also to giving opportunities to students to experience autonomy (Yackel & Cobb, 1996). Autonomy can be thought of in different dimensions. Along one dimension, at one end, one may have individual autonomy, while at another end, one may have social autonomy. In the context of PBL of mathematics, individual autonomy is enacted when learners strive to direct the course of their own work by relying on themselves to solve problems. Social autonomy on the other hand would entail scenarios where groups of learner’s bands together, working collaboratively to solve mathematical problems. In
particular, the concern in this article is with intellectual autonomy, which centers round learners experiencing a degree of liberty to direct their learning of mathematics. Giving learners choices to take greater control of their learning can empower them, especially in cases of students who have diverse backgrounds (Boaler & Staples, 2008).

There are a number of extensive studies that have emphasized meaning making in mathematics learning. These have often occurred by engaging learners with mathematical tasks of different levels of cognitive demand. The QUASAR project (Silver & Stein, 1996) engaged students and teachers in meaningful mathematics learning experiences that emphasized mathematical reasoning and thinking skills and problem solving. Students in QUASAR classrooms constantly had opportunities to work on challenging tasks that involved multiple representations, multiple solution strategies and in collaborative groups. This was in contrast to the traditional method of teaching mathematics, where students learn by memorizing and then practicing procedures. The findings from the QUASAR project are important because they demonstrate that the types of tasks that students work on are important to students’ learning outcomes. Students in QUASAR classrooms experienced the reform (hereafter, RF) approach to teaching, as opposed to the traditional approach (hereafter, TA).

Besides the QUASAR project, there are other studies which have foregrounded the value of promoting mathematics learning with different kinds of mathematical problems. In their study, Kolovou et al. (2007) also clearly link the notion of higher-order thinking with Stein et al. (2000)'s idea of levels of cognitive demand, where higher-order thinking is associated with a higher level of cognitive demand. As a result of analyzing mathematical tasks in Dutch textbooks, Kolovou et al. (2007) created a task classification system comprising three levels of tasks: straightforward, “gray area” and puzzle-like tasks. Straightforward tasks require only routine application of procedures to generate a solution whereas puzzle-like tasks may have more open-ended method approaches that may lead to a variety of solutions. Puzzle-like tasks require higher-order thinking, creativity and genuine problem solving. Gray-area tasks are neither straightforward nor puzzling but rather include some procedural aspects and some non-routine problem-solving aspects as well. Combinatorics is a field of mathematics concerned with counting. The subcategories for puzzle-like tasks are context and bare number problems, which are symbol-based tasks with little to no context. All in all, the approaches to studying mathematics in the QUASAR project as well as by Kolovou et al. (2007) encompass both constructivist and sociocultural approaches (Cobb & Yackel, 1996).

Although there is a strong background for studying mathematical tasks from the point of view of how cognitively complex the tasks are, given the mathematics incorporated in the tasks, and the demands of the tasks, there are other researchers who are increasingly drawing on tools such as the systemic functional grammar of Halliday, to understand and to reveal ways that language used in mathematics texts and tasks can help in both learning and in teaching of the subject. One such study is Herbel-Eisenmann (2007). In this study, the researcher operationalizes Halliday’s systemic functional grammar as a tool to study mathematics texts and tasks. In particular, Herbel-Eisenmann (2007) comprises the study of how to analyze mathematics texts and tasks with an aspect of grammar known as imperative clauses. In my research, I conceptualize imperatives in the same ways that Herbel-Eisenmann (2007) does, although Herbel-Eisenmann (2007) does not draw strongly on the analysis of imperatives in connection with agency, as I do. In connection with agency, I think of inclusive imperatives as affording students’ opportunities to exercise conceptual agency while exclusive imperatives give them opportunities to exercise disciplinary agency. My approach to modality is, however, different. Herbel-Eisenmann (2007) appears to conflate modalization and modulation (Halliday & Matthiessen, 2014), two aspects of modality. In their analyses of modality, Herbel-Eisenmann (2007) examine verbs such as “must,” and “will” that indicate modalization as well as adverbs such as “possibly” that indicate modalization under the overarching category of modality. In my research, I chose to have them separate in order to study how these two aspects of modality impact student positioning and the opportunity that positioning affords for agency and autonomy.

In this literature review, I have presented varying notions of agency and autonomy. I also defined intellectual, individual and social autonomy. I reviewed research on problem solving in mathematics education as it pertains to working in textbooks. The review also included examining the notion of open and closed task features and task types. I shall now proceed to methods incorporated in the study.

III. METHODS

For this study, I analyzed both the mathematical tasks in the textbook series, and the organizing mathematical texts that served as the corpus of each lesson text. From (Halliday & Matthiessen, 2014), we learn that texts can be analyzed linguistically on the basis of genres, stages, and moves. Examples of genres for mathematics texts are curriculum genres, expository genres, genres of written proofs, genres of summarized texts, and genres of research texts. This article is concerned in particular with curriculum genres. Curriculum genres can be thought of as a means of categorizing texts in terms of their social purposes. For the purpose of this article, I considered chapters of PBL textbooks as a curriculum genre. As chapters are composed of lesson sections, the chapter as curriculum genre comprises a collection of lessons as different curriculum stages. Each curriculum stage progresses the learning purpose of the chapter. In the same way that chapter genres have stages, each day’s lesson text can also be thought of as a genre onto itself. When the day’s lesson text is thought of as a genre, the chapter that comprises all the lesson texts can then be thought of as the microgenre. The lesson genre also has stages through which the lesson progresses. These stages are the main sections in which a lesson text is organized for work done during class time. For example, one textbook series has these main sections within each lesson text: orienting text, classwork, math notes. The other textbook...
series on the other hand is organized around the following main sections for work in each lesson text done during class time: interactive learning, guided instruction and practice, lesson check.

Apart from examining the stages in a given lesson text, communication from textbook authors to readers can also be examined at the level of a clause. A clause is a part of a sentence organized around a verb. According to (Halliday & Matthiessen, 2014), when two or more clauses are linked together, they form a clause complex. Often but not always, a single sentence with two or more clauses is an example of a clause complex. For example, the sentence, “When Dorothea came to my house, she had some pudding as she conducted research on the linguistic features of noun phrases” has three clauses “when Dorothea came to my house,” “she had some pudding,” and “as she conducted research on the linguistic features of noun phrases.” Clauses and clause complexes are important in this study because they are the main grammatical structures to which I shall apply linguistic analyses using tools based on Halliday's systemic functional grammar.

Clauses and clause complexes encapsulate the language moves that authors of mathematics texts make as they communicate their intents in the texts they generate. In the arena of mathematics learning, textbook authors make moves through the language clauses they use, as they communicate information or interact with students. A move consists of a clause or a set of clauses in a sentence. Moves can take the form of giving students information about how to solve a problem, asking students questions to check their understanding, or directing students during their work. I am thinking of the textbook authors’ moves as being realized through clauses and clause complexes. The Fig. 1 below shows the nested nature of curriculum genre, stage, and clauses as moves:

Fig. 1. Nested nature of categories within a lesson genre.

As the reader can easily notice, there is a nested quality to moves, stages and genres of a mathematics text. Moves occur within mathematics texts, which can be organized in different stages that are typically sections meant to advance the purposes of the text. Moves and the stages they are encapsulated in are in turn embedded with genres of mathematics texts. This multilayered approach to parsing mathematics texts offers opportunities to analyze genres of mathematics texts at different levels of abstraction and grain sizes. Within the context of this study, the purpose of categorizing a lesson text into stages and moves is to be able to conduct textual analysis at different grain sizes in order to draw out meaning in relation to opportunities for students to experience agency and autonomy. Analysis at different grain sizes is similar to an approach taken by Morgan in Morgan (2016). In the analysis conducted for this study, the focus of analysis will be on the grain size of a clause, which I selected as the unit of analysis. Analysis at the clause level is meant to support textual comparisons at the stage level.

Analyses based on systemic functional grammar, entail experiential (sometimes also referred to as ideational), interpersonal and textual functions. The experiential or ideational function involves the way text communicates ideas and processes going on in the world. The interpersonal function involves how text communicates interactions between individuals. The textual function involves the way text is organized. In this article, I draw particularly on the interpersonal function for clausal analysis, however analysis at the level of stages involves the textual function. Analyses and insights based on texts organized into stages that inform mathematics teaching alignments toward traditional or reform approaches found in the discussion section entail the ideational or experiential function. Analyzing the interpersonal function in lesson texts can help make apparent the power relations in the communications and interactions between textbook authors and students. These power relations involve whether or not the textbook authors grant students opportunities to exercise agency and autonomy in the extent to which they give students choices to take control of learning. With the interpersonal function, I shall draw on the notions of mood to show how textbook authors communicate information or interact with the reader through clause moods. This is one aspect of why the interpersonal function is useful in discerning meanings from a linguistic perspective in the way language is used to communicate in mathematics texts. The other aspect of this study in relation to the interpersonal function is modality, which shall also be discussed.

I categorize moves textbook authors make to communicate information and to enact relationships through language. The information communicated is in the form of clauses and clause complexes. Thus, the moves that textbook authors make depend on the kinds of clauses and clause complexes they draw on. In order to analyze clauses and clause complexes as moves, I analyzed the mood of each clause or clause complex. The mood of a clause or clause complex refers to whether it is declarative (which usually functions as a statement, in speech or written text), imperative (which usually functions as a command) or interrogative (which usually functions as a question). An example of a declarative mood of a clause is the statement “A function is given a name, that can be a letter, such as for g.” An example of an imperative mood of a clause is the command “Examine the input (x) and output (y) values in the table below” and an example of an interrogative mood of a clause is the question “Is there a relationship between the input and output values?” I should point out that although clauses in the three moods are associated with congruent speech functions, there can be cases where a clause may be associated with a non-congruent mood. Mood and speech functions are said to be congruent when the mood of a clause as determined by grammar matches up with what the clause is used for, as reflected in the speech function of the same clause. For example, the clause “Is there a relationship...
between the input and output values?” has an interrogative mood with question as its congruent speech function. There are however also cases of spoken English where there is a mismatch between the mood of a major clause and the corresponding usual speech function that matches up with the mood. These are instances of non-congruent alignment of mood and speech function.

For example, a clause that ordinarily would function as a question, presented in the interrogative mood in the congruent case could function as a command (typically presented in the imperative mood) instead. The following example shows the non-congruent case: “Can you indicate in your answer all the various ways that you think the problem can be solved?” In this example, although the clause complex ends with a question mark, and one would think that the student could simply answer with a “yes,” what students are really being directed to is to come up with all the methods they can think of for the mathematical task. In essence, they are being directed or commanded to produce a certain solution, for which only answering the question in the affirmative is not sufficient. Speaking in these kinds of ways where a clause functions in a non-congruent mood reflects some common ways that English is spoken in this (Western) culture. Such ways of communicating and interacting that involve non-congruent mood-speech function associations can appear in the articles of mathematics journals, in mathematics monographs and even in mathematics textbook authors’ communications and interactions with students. In this study, the majority of clauses are congruent with respect to mood and speech function. In the few exceptions where this is not the case, I use the speech function as the determining factor for assigning a role in regard to agency and autonomy.

Clauses in the declarative mood typically give students information that could be theory or fact about mathematics. Depending on the context of clauses in the declarative mood, students can use information in them to exercise either conceptual or disciplinary agency. With respect to clauses in the declarative mood, I focus on another feature of clauses that presents interpersonal meaning: modality. Modality refers to the degree of obligation or probability in a clause. Modality has two aspects: modalization and modulation. Briefly, modalization refers to the level of probability indicated in a clause. This can be judged by the presence of words in a clause such as “may” or “might” that lessen the authoritative nature of the clause. For example, a clause may read, “You may want to explore the function using a graph or with another method.” In this clause, students are positioned to choose what they intend to consider in order to explore a function. They may thus have a chance to exercise conceptual agency and intellectual autonomy in the process. Modulation on the other hand presents meaning related to obligation in the clause. Words such as “must” or “should” present high obligation. A clause that reads “you must check your solution by inserting the roots of the equation into the function” communicates high obligation on the part of students. On the other hand, words such as “can” indicate low obligation. This can be shown in a sentence such as “you can use a table to help plot the graph.” So, there is low obligation. In clauses with modulation, students are directed by the textbook authors.

This restricts students’ autonomy and ability to exercise conceptual agency to varying degrees.

Clauses in imperative moods typically demand action. For example, the imperative clause “Examine the input (x) and output (y) values below” demands action. For clauses in the imperative mood, I focus on two kinds: inclusive and exclusive imperatives. Inclusive imperatives can position students to exercise conceptual agency and intellectual autonomy in terms of giving students opportunities to think and to offer justifications for their own ideas. They include the use of verbs such as “explain,” “justify,” and “predict.” Exclusive imperatives on the other hand can position students to exercise disciplinary agency and little opportunity for intellectual autonomy. They involve positioning students to carry out standard mathematical procedures, and include verbs such as “find,” “solve,” and “calculate.”

Beyond the clausal level, the analysis of tasks and texts can be separated by the stages that each text is organized into. The genre level nests the stage level, which are the divisions or sections of a lesson text.

**TABLE I: DEGREES OF OPENNESS OF TASKS**

<table>
<thead>
<tr>
<th>Procedures without connections (Pw/cC)</th>
<th>Goal</th>
<th>Method</th>
<th>Complexity</th>
<th>Solution</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>closed</td>
<td>closed</td>
<td>closed</td>
<td>closed</td>
<td>closed</td>
<td>closed</td>
</tr>
<tr>
<td>Procedures with connections (Pw/C)</td>
<td>closed</td>
<td>closed</td>
<td>open</td>
<td>open</td>
<td>sometimes</td>
</tr>
<tr>
<td>Guided Real-Life (GRL)</td>
<td>closed</td>
<td>closed</td>
<td>open</td>
<td>open</td>
<td>closed</td>
</tr>
<tr>
<td>Investigative (GI)</td>
<td>closed</td>
<td>closed</td>
<td>open</td>
<td>open</td>
<td>open</td>
</tr>
<tr>
<td>Guided Synthesis (SyN)</td>
<td>closed</td>
<td>closed</td>
<td>open</td>
<td>open</td>
<td>open</td>
</tr>
<tr>
<td>Conceptual Explanations (CE)</td>
<td>closed</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>closed</td>
</tr>
<tr>
<td>Problem-Solving (PS)</td>
<td>closed</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>closed</td>
</tr>
<tr>
<td>Investigative (I)</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>close</td>
</tr>
</tbody>
</table>

Apart from the general use of language in each lesson text, the rich variety of tasks appearing in the two textbook series offer a range of opportunities for exercising agency and autonomy as they work on the tasks. It is important to point out that the tasks in Table I are in a hierarchy of degrees of openness, from the least to the most open task types. In other words, procedures with connections tasks have a higher degree of openness than procedures without connections tasks, because the solution task feature can be open in the former and not in the latter. Likewise, guided investigative tasks have a higher degree of openness than procedures with connections tasks, because the extension task feature can be open in the former and not in the latter. Overall, procedures without connections tasks have the lowest degree of openness whereas investigative tasks have the highest degree. Each of these tasks gives students...
opportunities to develop different forms and extents of agency and autonomy. For instance, procedures without connections tasks give students opportunity to develop disciplinary agency. Procedures with connections tasks afford students the opportunity to develop disciplinary and conceptual agency as well as the reactive case of intellectual autonomy. This is due to the open nature of the solution variable, which allows students to explain their thinking. It is also due to the open nature of the complexity variable, which means that students will have to make connections to figure out some of the gaps in the method needed to execute procedures. Guided investigative tasks allow students to develop conceptual agency and reactive autonomy. And so on. All the tasks give students different opportunities.

Where a task had subsections, I coded those subsections as closed or open in terms of each of the five task features (goal, method, complexity, solution, and extension). I then selected the most open state for each task feature across the entire task to decide on task type. Each task type offers students opportunities to develop agency and autonomy based on which task features it has open. Thus, Table I begins with task features and ends with task types and the opportunities they align with for agency and autonomy. An example of a task analyzed for this study is the following: “How are the equations of \( y = \frac{-2}{3} b^2 \) and \( y = \frac{-2}{3} b^2+3 \) similar? How are they different?”

IV. RESULTS

The results of this study shall be presented as they pertain to the textual, the ideational and the interpersonal functions of text. The most consequential result connects to the textual function. It involved the manner of use of exclusive and inclusive imperatives across stages in the lesson texts of RF and TA textbooks. Exclusive imperatives may prompt learners to follow a specific action. An example of such an exclusive imperative is the word ‘multiply’. Examples of other exclusive imperatives resulting from analyses are calculate, color, draw, evaluate, expand, find, graph, label, locate, mark, record, rewrite, round, sketch, simplify, solve, write. Inclusive imperatives may prompt learners to draw on their own resources in order to satisfy the statute of the task. An example of one such inclusive imperative is the word ‘investigate’. Examples of inclusive imperatives from the analyses that students draw on their own thinking are: Compare, confirm, conjecture, create, describe, design, explain, explore, generate, justify, predict. Table II shows distributions of exclusive and inclusive imperatives employed at different stages of RF and TA lesson texts:

<table>
<thead>
<tr>
<th>TABLE II: RESULTS FOR 18 RF AND 18 TA LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stages</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>RF</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

A close look at Table II reveals that the RF and TA lesson texts each had comparable levels of inclusive imperatives although TA textbooks had higher frequencies of exclusive imperatives compared to the RF textbooks. The latter case is because the TA textbook learning orientation emphasizes repetition and practice of basic methods. Inclusive imperatives on the other hand give students the chance to include more of their own thinking, giving students opportunities to develop conceptual agency and intellectual autonomy. In this regard, the RF and TA textbooks had comparable frequencies of inclusive imperatives although the RF textbook had a greater variety than the TA textbook because the RF textbook emphasized mathematical investigation and exploration more than the TA textbook.

The reform and traditional approaches to learning are also reflected in the findings from the linguistic perspective. For the RF textbook lesson texts, all the mathematical tasks are located in one stage of the lesson text. The TA lesson texts on the other hand show mathematical tasks in each of three stages. These differences in where mathematical tasks appear in the lesson text reflect actual classroom process. This means that in classrooms where the RF textbook is used, students solve tasks during one continuous lesson session. In classrooms where the TA textbook is used, students solve tasks in three sessions of a lesson corresponding to the three stages in the lesson text. Analysis of inclusive and exclusive imperatives showed that RF textbook lessons had roughly the same number of inclusive and exclusive imperatives in the second stage of the lesson text. This means that all work on tasks happens during this one stage. In the TA textbook lesson texts, the arrangement is different. There are about five times as many inclusive imperatives as exclusive imperatives in stage one, about five times as many exclusive imperatives as inclusive imperatives in stage two and about twice as many inclusive imperatives in stage three as exclusive imperatives. This is an interesting finding because what it shows is that the first and third stages of TA textbook lesson texts have more challenging mathematical tasks. The second stage on the other hand has more procedural tasks. Therefore, the TA textbook lesson texts start and end the lesson mostly with challenging mathematical tasks. The main section of the lesson has mostly procedural tasks, during which students work on more basic mathematics that reinforces procedural skills. This lesson structure aligns more with a traditional approach to instruction, whereas the RF lesson texts where all learning occurs during one continuous session conforms with a reform approach to instruction.

V. DISCUSSION

As presaged in the introduction, this article is based on the notion that words alone can and should be sufficient in conveying mathematical ideas. We have seen in this study that analyses of the stages in the mathematics lesson texts from two comparable textbook series revealed that the use of clauses within these stages reflected the teaching orientations that are philosophical underpinnings guiding these textbook series. In this regard, the textual function of systemic functional grammar (Halliday & Matthiessen, 2014) is highlighted. The textual function pertains to the organization of text. The manner in which TA and RF texts are organized into sections together with the use of
exclusive and inclusive imperatives in each section indicate the intentions of the textbook authors in regard to how lessons should proceed based on each category of lesson text. The lesson texts from RF textbooks are organized so that students work collaboratively among themselves, with the teacher interjecting or intervening on occasion to provide some guidance. The lesson texts from TA textbooks on the other hand foreground a more active teacher-led role. Taken together, these stages allow lesson texts to be analyzed section by section, not only at the level of moves (that is, of imperative clauses as an example, as demonstrated in Herbel-Eisenmann, 2007) but now also at the level of stages and genres of texts. Herbel-Eisenmann (2007)’s initial analysis of lesson texts encompasses linguistic analysis of the mood of entire texts, with subsequent focus shifting to analysis of mathematical tasks within texts. The conceptual framework I have adopted in the present work allows for an added dimension of intermediate grain size analysis (that is, between a micro level analysis of mood demonstrated at the clausal level, and a macro level analysis of text genres). This intermediate grain size analysis of texts serves the function of revealing progressions within lesson texts which may appear to conform with the philosophical orientations upon which the texts are based. Such analyses can serve as benchmarks in researcher and in practitioner communities of how closely mathematical texts align with the philosophical orientations of teaching mathematics upon which those texts are based. These two philosophical orientations, one where students work mostly by themselves and the other where the teacher plays a more active role in students’ learning are simply different approaches to teaching and learning mathematics. What is important are the opportunities each learning approach offers students to develop forms of agency and autonomy. That RF textbooks gave students more opportunities to work on challenging tasks aligns with their philosophy of allowing students to be introduced to mathematical ideas that will engage them and possibly cause them to struggle as they work individually or in groups to solve those tasks. RF textbooks support complex instruction, a version of reform mathematics where students collaborate on mathematics tasks. This approach to learning bridge’s ability levels and different backgrounds. TA textbooks on the other hand support a different philosophy, a more traditional approach. Rather than having learning mathematics collaboratively, sometimes even through struggle, TA textbooks positioned students to learn mathematics first by working through example tasks led by a teacher and then by individually practicing on simple, similar tasks. For lesson texts in TA textbooks, as shown by the analysis, the teacher plays an important role in leading learning. Rather than having the burden for learning fall more heavily on the student, in TA textbooks it falls more heavily on the teacher. In other words, getting all students to achieve a basic level of understanding by practicing examples and solving simple tasks to reinforce learning is the role of the teacher. These two approaches to learning, one student led, and teacher led learning, serve as a distinct difference between the two-textbook series.

As such, the main significance of the results portrayed in the present work is a realization that the organization of lesson texts in textbooks can reflect the philosophical orientations of reform and traditional approaches to teaching. In this regard, the ideational or experiential function of systemic functional grammar is brought to the fore. The experiential or ideational function involves the way text communicates ideas and processes going on in the world. Here, the ideational aspects are the orientation toward either reform or traditional pedagogical notions and practices. It is an important finding because it brings attention to yet another role that an understanding of linguistics can play in the teaching and learning of mathematics.

The third aspect of systemic functional grammar, the interpersonal function, also features significantly in the results. For this third aspect, the more important result pertains to differences in the use of declarative clauses with modulation in the two-textbook series. In other words, it pertains to mood, and in particular to the modulation aspect of mood. Modulation has to do with the degree to which a clause indicates obligation. We find in the lesson texts from TA textbooks that declarative clauses with modulation feature strongly in the main section of the lesson text, that is, the section titled “Guided Instruction & Practice.” This stage of the text corresponds to the main lesson module where a teacher guides students by demonstrating examples and having students practice by following the teacher’s lead. The lesson texts from RF textbooks on the other hand employ declarative clauses with modulation primarily in the earliest stage of each lesson text, the LO, or “Lesson Orientation” stage. This is the stage where the lesson text outlines lesson goals or objectives. The RF text thus, through the lesson text, obligates learners to pursue specific goals or objectives for each lesson period. Outside of this, use of declarative clauses with obligation is left to a minimum in the main stage of their lesson texts. This orientation aligns with the orientation of complex instruction, wherein learners primarily lead themselves rather than experiencing obligation from an external authority such as the textbook or the classroom teacher.

Starting with the works of George Pólya, the study of mathematical problem solving has had a distinguished pedigree (c.f., Schoenfeld, 1985). Within mathematics teaching, the study of language used for mathematics instruction in pre-university textbooks as for instance shown in Herbel-Eisenmann (2007) has gained increasing importance because it is now understood that language is a primary conveyor of mathematics content and context, such as we see in Wijaya et al. (2015) and Kilpatrick et al. (2001). This is why mathematics at the high-school level evolved to include linguistics. The approach to analyzing language in mathematics texts as presented in this article can be a powerful way to study a plethora of genres of texts containing mathematics contents and contexts (c.f. mathematics journals, monographs, teaching notes) to ascertain degrees to which the contents and contexts of these texts reflect or support more formal mathematics symbols in order to communicate conjectures, exercises, proofs, theorems and such. This is the primary affordance of the method for analyzing texts given in the present work.
VI. CONCLUSION

For more than four decades, the mathematics education research community has extensively researched problem solving in mathematics teaching and learning (Lester, 2013; Silver et al., 2005). Research into mathematical problem solving continues unabated (Yeo, 2017a; Yeo, 2017b; Silver, 2013). It is clear from research literature (Lester & Cai, 2016) that much energy and effort has gone into studying mathematical problem solving as an activity that can be taught. There has also been well established work (Silver & Stein, 1996; Stein et al., 2000; Stanic & Kilpatrick, 1989) that has matured from research into practice for teaching mathematical problem solving in the curriculum. There now exists traditions for teaching mathematical problem solving in pre-university curricula. Two of those traditions, as discussed in the present work, are further expounded upon in Sood and Jitendra (2007).

ACKNOWLEDGMENT


CONFLICT OF INTEREST

The author declares that he does not have any conflict of interest.

REFERENCES


